Oligopoly
Industrial Organization

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Outline

Introduction

Definition of an oligopoly

An industry or a market in which a small number of firms compete.

- Most markets fit this description: telecommunications, software industry, but also mineral water industry, ...
- In an oligopolistic market, a firm cannot ignore the behavior of its competitors ...
- ... and the reaction of competitors to its own decisions
- The **theory of oligopoly** aims at studying these **strategic interactions**

Note: oligopoly with 2 firms = duopoly

Cournot model

- developed in 1838 by French mathematician and economist Augustin Cournot
- two firms produce identical (homogeneous) goods ("perfect substitutes") and compete in quantities
- the price is set so that the total quantity of goods produced is sold → the market price is $p = P(Q)$, where $Q = q_1 + q_2$ is the total quantity produced
- the marginal cost, c, of producing each unit of the good is assumed to be constant and identical for both firms
- the profit function of firm $i \in 1, 2$ is then

$$\Pi_i = \big(P(Q) - c\big) q_i$$

- for simplicity, we assume that $P(Q) = 1 - Q$

Cournot model

- each firm i sets its quantity q_i to maximize its profits, given the quantity q_j set by the other firm (so, q_j is considered *fixed*)
- the **first-order condition** of the maximization problem for firm i is

$$1 - (q_1 + q_2) - c - q_i = 0$$

- we are looking for a **symmetric equilibrium** such that $q_i = q$. So:

$$q^\star = \frac{1-c}{3}$$

and

$$\Pi^\star = \frac{(1-c)^2}{9}$$

Cournot model

The equilibrium profit is:

$$\Pi^\star = \frac{(1-c)^2}{9} > 0$$

In Cournot competition, firms make profits!

However, the Cournot model seems somewhat unrealistic:

- Few examples of markets where firms set quantities rather than prices
- We don't know well how the price is set in the market

A model where firms set prices instead of quantities? $\rightarrow$ Bertrand model

Bertrand model

- Developed in 1883 by the French mathematician and economist Joseph Bertrand
- Two firms, 1 and 2, produce identical goods (*perfect substitutes*) and compete in **prices**
- The demand function is $q = D(p)$
- The marginal cost of production, c, is constant and identical for both firms
- We assume (not crucial) that the market is split evenly if firms offer the same price:

$$D_i(p_i, p_j) = \begin{cases} D(p_i) & \text{if } p_i < p_j, \\ \frac{1}{2} D(p_i) & \text{if } p_i = p_j, \\ 0 & \text{if } p_i > p_j. \end{cases}$$

- The profit function of firm i is:

$$\Pi_i(p_i, p_j) = (p_i - c)\, D_i(p_i, p_j)$$

Bertrand equilibrium

We look for the Nash equilibrium for this one-stage game.

Bertrand paradox

The game has a unique Nash equilibrium such that firms set $p_1^* = p_2^* = c$.
At the equilibrium, we have $\Pi_1^* = \Pi_2^* = 0$ and $W = W^*$ (social welfare is maximized).

A strong result:

- As the number of firms increases from one (monopoly) to two (duopoly), the price falls from the monopoly price to the competitive price and stays at the same level as the number of firms continues to increase.
- Two firms are sufficient to reach a perfectly competitive equilibrium.
- Because this result is extreme and generally inconsistent with reality, it is called the "Bertrand paradox".

Proof

If $p_1 > p_2 > c$?

Then, firm 1 increases its profit by setting $p_1 = p_2 - \varepsilon$

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$$\frac{D(p_1)(p_1 - c)}{2} < D(p_1 - \varepsilon)\,(p_1 - c - \varepsilon).$$

If $p_1 > p_2 = c$?

Then, firm 2 increases its profit by setting $p_2 = p_2 + \varepsilon$

An example: the NYC pizza war

link to full story in NY Times (September 5, 2012)

Bertrand competition with different marginal costs

Let us assume that $c_1 < c_2$. We denote by $p^m(c)$ the monopoly price for marginal costs c.

If costs are close enough: $c_1 < c_2 < p^m(c_1)$

- There is a unique Nash equilibrium with $p_1^* = c_2 - \varepsilon$ and $p_2^* = c_2$
- Only firm 1 makes profit:

$$\Pi_1^* = (c_2 - c_1)D(c_2) \quad \text{and} \quad \Pi_2^* = 0$$

If firm 1 is much more efficient than firm 2: $c_1 < p^m(c_1) < c_2$

- There is a unique Nash equilibrium with $p_1^* = p^m(c_1)$ and $p_2^* = c_2$
- Only firm 1 makes profit:

$$\Pi_1^* = \Pi_1^m \quad \text{and} \quad \Pi_2^* = 0$$

Solutions to the Bertrand paradox

Four important solutions to the "Bertrand paradox", corresponding to four important assumptions on which the model is based:

1. Products are homogeneous
2. Short-run competition (a static analysis of a one-stage game)
3. Firms have no capacity constraints
4. Consumers are perfectly informed

Relaxing one of these assumptions solves the paradox:

1. Product differentiation
2. Dynamic competition (repeated interactions)
3. Firms are capacity constrained
4. Imperfect information

Product differentiation

For example, let us assume a **geographic differentiation**:

- Two ice-cream vendors, 1 and 2, are located at both ends of a beach.
- If $p_1 = c$, is $p_2 = c + \varepsilon > c$ possible?
- Consumers close to vendor 2 may prefer to buy at a slightly higher price from 2 rather than walking to 1.
- Horizontal and vertical differentiation theory: the Hotelling model

In the presence of product differentiation, $p_i > c$ can be an equilibrium.

$\rightarrow$ See lecture on **differentiation**.

Dynamic competition

- The Bertrand model assumes that firms compete only during a single period.
- Starting from $p_1 = p_2 > c$, a firm has strong incentives to undercut its price.
- In a more dynamic framework, what can happen?
- Firms take into account the effect of their price cut on rivals' behavior in future periods.
- If there is a *punishment*, firms compare the short- and long-term gains of a price war.

In a repeated interaction framework, $p_i > c$ can be an equilibrium.

$\rightarrow$ See lecture on **collusion**.

Capacity constraints

- The Bertrand model assumes that firms do not have capacity constraints.
- If $p_1 = p_2 = c$, the two firms share the demand: $\frac{D(c)}{2}$.
- If firm 2 increases its price slightly, $p_2 = c + \varepsilon$, we have assumed that firm 1 meets the demand, that is, $D(c)$.
- But firm 1 may not be able to satisfy all of the demand: capacity constraints
- In this case, firm 2 can increase its price and still keep some of the market.

If firms face capacity constraints, $p_i > c$ can be an equilibrium.

$\rightarrow$ See below.

Imperfect information

With imperfect information, $p_i > c$ can be an equilibrium.

- The same product is sold in many different stores.
- Consumers are not informed about prices.
- There is a cost ε to go from one store to another (*search cost*).
- If $p_1 = p_2 < p^m$, then a profitable deviation is possible to $p_1 + \varepsilon$.

Price competition with capacity constraints

We are going to study a model of price competition where firms have capacity constraints.

We consider the following model:

- Two firms compete in prices, but are capacity-constrained.
- The demand function is linear:

$$D(p) = 1 - p.$$

- Hence, the inverse demand is:

$$p = P(q_1 + q_2) = 1 - (q_1 + q_2).$$

- Firm i cannot produce more than its production capacity $\bar{q}_i$. Therefore,

$$q_i \leq \bar{q}_i.$$

- The unit cost of acquiring capacity $\bar{q}_i$ is $c_0 \in [\frac{3}{4}, 1]$.
- There are no production costs ($c = 0$).

Price competition with capacity constraints

- We need to define a rationing rule.
- Determines which consumers will be served if a firm cannot meet all demand.
- "Efficient" rationing rule: *consumers with higher willingness to pay are served first*
- If $p_1 < p_2$ and $\bar{q}_1 < D(p_1)$:

$$\tilde{D}_2(p_2) = \begin{cases} D(p_2) - \bar{q}_1 & \text{if } D(p_2) \geq \bar{q}_1, \\ 0 & \text{if } D(p_2) < \bar{q}_1. \end{cases}$$

- Another possible rule: the "proportional" rationing rule.

Results

We obtain the following result:

Price competition with capacity constraints

The unique Nash equilibrium is such that firms set the same price:

$$p^* = 1 - \left(\bar{q}_1 + \bar{q}_2\right)$$

Consequence: at the stage of defining their production capacities $\bar{q}_i$, firms' gross profits equal:

$$\Pi_i^g = \left[1 - (\bar{q}_i + \bar{q}_j)\right] \bar{q}_i$$

→ That is the profit function in the **Cournot model**.

Proof: preliminaries

First, given that $c_0 \in [\frac{3}{4}, 1]$, what is the upper bound for $\bar{q}_i$?

To answer this question, what is the maximum profit for firm i?

$\rightarrow$ It is the monopoly profit!

As $D(p) = 1 - p$ and $c = 0$, the monopoly price is equal to $p^m = \frac{1}{2}$.

So the monopoly profit is:

$$\Pi^m = \frac{1}{4} - c_0 \bar{q}_i.$$

Therefore,

$$\bar{q}_i \leq \frac{1}{3}$$

because we must have

$$\frac{1}{4} - c_0 \bar{q}_i \geq 0$$

Proof of existence

Now, let us show that

$$p^* = 1 - \left(\bar{q}_1 + \bar{q}_2\right)$$

is a Nash equilibrium.

Observation: $p^* > c$ because

$$\bar{q}_1 + \bar{q}_2 < \frac{2}{3}$$

Can firm i lower its price?

No. It would not increase its profits, as it is already producing at maximum capacity.

$\rightarrow$ "Bertrand competition" does not work because firms face capacity constraints.

Proof of existence

If firm i sets a higher price $p > p^*$:

- Firm j captures the entire demand up to the maximum of its production capacity.
- A fraction of demand is not satisfied: we call it the residual demand.
- The residual demand is equal to
$$1 - p - \bar{q}_j$$
- Hence, firm i makes profit:
$$p\left(1 - p - \bar{q}_j\right) = (1 - q - \bar{q}_j)\, q$$
- This is the Cournot profit. It is concave in q, and its derivative with respect to q evaluated at $q = \bar{q}_i$ is:
$$1 - 2\bar{q}_i - \bar{q}_j \geq 0 \quad \text{since } \bar{q}_i, q_j \leq \frac{1}{3}$$
- Therefore, the optimal value of q must be $q = \bar{q}_i$

Proof of uniqueness

$$p^* = 1 - (\bar{q}_1 + \bar{q}_2) \text{ is the } \textbf{unique} \text{ Nash equilibrium.}$$

- $p_1 = p_2 = p > P(\bar{q}_1 + \bar{q}_2)$ is not an equilibrium because at least one firm does not reach its maximum production capacity. Hence, it can profitably lower its price.
- $p_1 = p_2 = p < P(\bar{q}_1 + \bar{q}_2)$ is not an equilibrium. By setting $p_i = p_i + \varepsilon$, the firm would sell the same quantity (its production capacity) at a higher price.
- $p_1 < p_2$ is not feasible, because firm 1 is encouraged to increase its price.

General result

Consider the following two-stage game:

- First, firms simultaneously choose their production capacities.
- Second, firms observe the chosen capacities and simultaneously choose their prices.

Kreps and Scheinkman (1983): capacity + price = quantities

If the demand function is concave and we use the *efficient rationing rule*, then the equilibrium of this two-stage game is equivalent to the equilibrium of **Cournot competition** (competition in quantities).

Cournot or Bertrand competition?

Which model is more appropriate if production capacity is easily adjustable?

Rule of thumb

If the production capacity can be easily adjusted, the **Bertrand model** is a better representation of duopoly competition. Otherwise, if it is difficult to adjust production capacity, the **Cournot model** is more appropriate.

Examples of markets where production capacity is difficult to adjust: → markets for physical goods (car manufacturers, airplanes, cement, . . .)

Examples of markets where production capacity is easy to adjust: → service markets (banking, insurance, . . .)

Content industries: Bertrand or Cournot?

In the **content industries** (music, movies, video games, . . .), we have observed an evolution of the distribution model:

- *old distribution model*: sale of content on physical devices (CDs, DVDs, cartridges)
- *new distribution model*: sale of digital content via online stores (e.g., iTunes) or Stream

Which competition model better represents the old distribution model?
And which the new one?

What consequences can we predict from this change in the competition model?

Cournot competition with n firms

- Consider a market with the linear demand

$$D(p) = 1 - p$$

- n firms operate in this market and produce a homogeneous good
- They compete in **quantities** (Cournot competition)
- The cost function of firm i with constant marginal cost c is

$$C_i(q_i) = cq_i$$

- What is the equilibrium price?
- What is the equilibrium profit?

Cournot competition with n firms

- The inverse demand is $P(Q) = 1 - Q$, where $Q = \sum_{i=1}^{n} q_i$
- The profit function of firm i is $\Pi_i = \big(P(Q) - c\big) q_i$
- Firm i chooses its quantity q_i to maximize its profit
- The first-order condition of the maximization problem is

$$1 - Q - c - q_i = 0$$

- We look for a symmetric equilibrium with $q_i = q$. We find

$$q^* = \frac{1-c}{n+1}, \qquad \Pi^* = \frac{(1-c)^2}{(n+1)^2}$$

$\rightarrow$ the greater the number of firms, the lower the Cournot profit!

Comparison in terms of market power

Assumptions:

- Let us assume there are n firms with the same marginal cost of production c.
- We define the **Lerner index** for firm i as

$$L_i = \frac{p_i - c}{p_i}$$

Comparison: monopoly, Bertrand, Cournot

- In a monopoly, we have:

$$L_i = \frac{1}{\varepsilon}$$

- In Bertrand competition, we have:

$$L_i = 0$$

- In Cournot competition, we have:

$$L_i = \frac{\alpha_i}{\varepsilon},$$

where α_i is the market share of firm i

Proof

- $P(Q)$ represents the inverse demand function, with $Q = q_i + q_j$ total quantity produced
- Then, firm i's profit function is

$$\Pi_i = \big(P(q_i + q_j) - c\big)\, q_i$$

- The first-order condition is:

$$P(q_i + q_j) - c + q_i P'(q_i + q_j) = 0$$

- So,

$$\frac{P - c}{P} = -\frac{q_i P'}{P} = -\frac{q_i}{Q}\frac{P'Q}{P} = -\frac{q_i}{Q}\frac{Q}{PD'}$$

- *i.e.*,

$$L_i = \frac{\alpha_i}{\varepsilon},$$

where $\alpha_i = \dfrac{q_i}{Q}$ is the market share of firm i and ε is the (absolute value of the) price elasticity of demand.

Merger in the Portuguese cement industry

- There are five main players producing homogeneous goods and competing in quantities:
  - Secil
  - Cimpor
  - LafargeHolcim
  - Cimentos Liz
  - Mota-Engil
- Let us assume there are rumore of mergers between Secil and Cimpor
- We build a simple competition model to study the incentives of Secil and Cimpor to merge
- Let q_i be the quantity of firm i and the total quantity

$$Q = q_1 + q_2 + q_3 + q_4 + q_5$$

- The demand function is

$$D(p) = 60 - p$$

- Assume c $= 0$

Merger in the Portuguese cement industry

1. **Determine the Nash equilibrium for a quantity competition game.** Compute the quantity and profit in equilibrium for each firm.

- The quantity in equilibrium for a Cournot model with n firms is:

$$q^*(n) = \frac{60}{n+1}$$

- The Cournot profit with n firms is:

$$\Pi_i^*(n) = \frac{3600}{(n+1)^2}$$

- Therefore, for $n = 5$, we have:

$$q^*(5) = \frac{60}{6} = 10 \quad \text{and} \quad \Pi_i^* = \frac{3600}{36} = 100$$

Merger in Cournot competition

2. We assume that firms 1 and 2 merge (Secil and Cimpor) to form a new firm, SC. Compute again the equilibrium, the quantity produced, and the equilibrium profit for firms SC, 3, 4, and 5.

- We move from an oligopoly with 5 firms to an oligopoly with 4 firms.
- Thus, for $n = 4$, we have:

$$q^*(n) = \frac{60}{5} \quad \text{and} \quad \Pi_i^* = \frac{3600}{25} = 144$$

Merger in Cournot competition

3. Are firms 1 and 2 better off if they merge? How do the profits of firms 3, 4, and 5 evolve?

- Secil and Cimpor are not well advised to merge because their total profit decreases:

$$144 < 2 \times 100.$$

- On the other hand, LafargeHolcim, Cimentos Liz, and Mota-Engil are better off:

$$144 > 100.$$

Merger in Cournot competition

- Let's consider a market with $n > 1$ firms.
- The marginal cost is constant and identical for all firms: c
- Linear demand: $P(Q) = a - bQ$
- In equilibrium, a firm makes the following profit:

$$\Pi_i^*(n) = \frac{1}{b}\frac{(a-c)^2}{(n+1)^2}$$

- If k firms merge, $n - k + 1$ firms will remain in the market.
- So, a merger involving k firms is profitable if

$$\Pi_i^*(n-k+1) \geq k\,\Pi_i^*(n)$$

Merger in Cournot competition

A merger is profitable only if it involves more than 80% of the firms in the market.

Merger in Bertrand competition

- Let us consider a market with $n > 1$ firms.
- The marginal cost is constant and identical for all firms: c
- In equilibrium, we have

$$p_1^* = \cdots = p_n^* = c$$

  and profits are equal to 0.
- If $k < n$ firms merge, how is the equilibrium changed?
- It is **unchanged**: the price remains equal to c and profits are still equal to 0.
- How much market share need to merge to make a merger profitable?

Merger in Bertrand competition

a merger is profitable only if it involves all (100%) of the firms in the market.

Conclusion on mergers

Merger decisions cannot be explained by incentives to reduce competition in the market alone!

Other dimensions to explain mergers?

- Synergies (e.g., cost reductions)
- Becoming an industry leader (the competition model is changing: Stackelberg instead of Cournot)
- Portfolio strategy: product line extension, economies of scale in sales and marketing, bundling, ...

Take-aways

- Bertrand competition between identical firms leads to marginal-cost pricing ("Bertrand paradox").
- The Bertrand paradox is no longer verified if we relax one of the four main assumptions.
- If capacity constraints can be easily adjusted in the short run, firms are more likely to compete à la Bertrand. If capacity is fixed in the medium run, firms compete à la Cournot.
- When n firms compete in Cournot, their profit is inversely proportional to the number of firms playing in the market.
- A merger is not always profitable in Cournot competition; it must involve at least 80% of the firms in the market. In Bertrand competition, it requires 100% of the firms in the market. Mergers cannot be explained by an incentive to reduce competition alone.

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